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By

I. G. Sevruk

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UNEDITED ROUGH DRAFT TRANSLATION

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On the Soleness of Solution of Basic Problem concerning Free Thermal Convection of Liquid.

by

I.G.Sevruk

1. Assuming that in a certain volume V, limited by a closed surface S is situated in state of thermal convective movement a viscous, mechanically incompressible liquid, and assuming that on the boundary of the zone and at the initial moment of time is given the distribution of velocity and temperature of the liquid. It is necessary to find the distribution of these values by the volume of the liquid at a given moment of time.

Assuming the existence of a solution for the mentioned problem, we will prove its soleness. For this we employ the D.Ye.Dolidze [1] method.

Equations of free thermal convection have the form of [2] :

$$\frac{\partial v}{\partial t} + (v\nabla)v = -\nabla\frac{p}{\rho} + \nu\Delta v - \beta g T, \tag{1}$$

$$\frac{\partial T}{\partial t} + (v\nabla T) = \chi\Delta T, \tag{2}$$

$$\operatorname{div} v = 0, \tag{3}$$

where v- velocity of the liquid, p-read from hydrostatic pressure of the liquid, T-temperature of the liquid, read from a certain constant mean value T^*, $\rho = \rho(T^*)$-density of the liquid, g-gravity acceleration, ν, β, χ - coefficients of kinematic viscosity of thermal expansion and heat conduction of the liquid.

The sought for values should satisfy the given conditions:

$$v_s = 0, \quad T_s = \varphi(r, t), \tag{4}$$

$$v_0 = f(r), \quad T_0 = \psi(r) \tag{5}$$

(the sign s designates the boundary, and 0 - the initial value of corresponding value).

2. We will assume, that there are two solutions for problem (1) - (5):

$$(v_1, p_1, T_1) \text{ и } (v_2, p_2, T_2). \tag{5a}$$

Then functions

$$u = v_1 - v_2, \quad q = p_1 - p_2, \quad \theta = T_1 - T_2 \tag{6}$$

will satisfy equations

$$\frac{\partial u}{\partial t} + (v_1\nabla)\, u + (u\nabla)\, v_2 = -\nabla \frac{q}{\rho} + \nu\Delta u - \beta g\theta, \tag{7}$$

$$\frac{\partial \theta}{\partial t} + (v_1\nabla\theta) + (u\nabla T_2) = \chi\Delta\theta, \tag{8}$$

$$\operatorname{div} u = 0 \tag{9}$$

and conditions:

$$u_s = 0, \quad \theta_s = 0, \tag{10}$$

$$u_0 = 0, \quad \theta_0 = 0. \tag{11}$$

To prove the theory of soleness it is necessary to show that $u \equiv 0$ and $\theta \equiv 0$.

For this purpose equation (7) is multiplied scalarly by u, equation (8)-by θ, we will combine same and then integrate by the volume of liquid; we will obtain

$$\int_V \left(u\frac{\partial u}{\partial t} + \theta\frac{\partial \theta}{\partial t}\right) dV + \int_V \{u\,(v_1\nabla)\,u + \theta\,(v_1\nabla)\,\theta\}\, dV + \int_V u\,(u\nabla)\, v_2 dV =$$

$$= -\int_V u\nabla\frac{q}{\rho}\, dV + \int_V (\nu u\Delta u + \chi\theta\Delta\theta)\, dV - \int_V \theta u\,(\beta g + \nabla T_2)\, dV. \tag{12}$$

Taking into consideration the continuum equation and the boundary values of the functions u and θ, we transform the integrals included in (12) with the aid of the Gauss-Ostrogradskiy and Grin formulae. In consequence we will obtain

See Page 2a for Eq. 13

$$\int_V \left(u\frac{\partial u}{\partial t}+\theta\frac{\partial \theta}{\partial t}\right)dV=\frac{d}{dt}\int_V \frac{1}{2}(u^2+\theta^2)\,dV=\frac{1}{2}\frac{dK}{dt},$$

$$K=\int_V (u^2+\theta^2)\,dV=K(t);$$

$$\int_V \{u(v\nabla)u+\theta(v\nabla)\theta\}\,dV=\int_V (v\nabla)\left(\frac{u^2}{2}+\frac{\theta^2}{2}\right)dV=$$

$$=\frac{1}{2}\int_V \operatorname{div}[v(u^2+\theta^2)]\,dV=0,$$

$$\int_V u\nabla\frac{q}{\rho}\,dV=\int_V \operatorname{div}\left(u\frac{q}{\rho}\right)dV=0,$$

$$\int_V (\nu u\Delta u+\chi\theta\Delta\theta)\,dV=-\int_V \{\nu(\operatorname{rot} u)^2+\chi(\operatorname{grad}\theta)^2\}\,dV.$$

Eq. 13

Substituting the found values of integrals in (12) we will obtain

$$\frac{dK}{dt} = -2\int_V \{\nu(\mathrm{rot}\, u)^2 + \chi(\mathrm{grad}\,\theta)^2\}\, dV - 2\int_V u(u\nabla)v_2 dV - \\ -2\int_V \theta u(\beta g + \nabla T_2)\, dV. \qquad (14)$$

Since $\nu, \chi > 0$, then the first component of the right part (14) is negative and, consequently

$$\frac{dK}{dt} \leqslant -2\int_V u(u\nabla)v_2 dV - 2\int_V \theta u(\beta g + \nabla T_2)\, dV. \qquad (15)$$

In the initial moment of time the function K(t) equals zero, and further on it cannot be negative. Consequently in a certain interval of time $0 \leqslant t \leqslant \tau$

$$\frac{dk}{dt} \geqslant 0.$$

In such a case, by comparing the absolute values of left and right parts of the inequality (15) for $0 \leqslant t \leqslant \tau$ we obtain

$$\frac{dK}{dt} \leqslant 2\left|\int_V u(u\nabla)v_2 dV\right| + 2\left|\int_V \theta u(\beta g + \nabla T_2)\, dV\right|. \qquad (16)$$

Evaluating the magnitude of the integrals, included in (16), we obtain

$$\left|\int_V u(u\nabla)v_2 dV\right| \leqslant M\int_V u^2 dV \leqslant M\cdot K, \qquad (16a)$$

where M - mean value $\left|l\,\frac{dv_2}{\partial l}\right|$ (l - crosscut of direction u);

$$\left|\int_V \theta u(\beta g + \nabla T_2)\, dV\right| \leqslant N\int_V |\theta|\cdot|u|\, dV \leqslant N\int_V \frac{1}{2}(u^2+\theta^2)dV = \frac{1}{2}N\cdot K, \qquad (16b)$$

where N - mean value $\left|\beta g l + \frac{\partial t_2}{\partial l}\right|$.

Taking into consideration the obtained evaluations, we will have for $0 \leqslant t \leqslant \tau$:

$$\frac{dK}{dt} \leqslant A\cdot K, \qquad (17)$$

where A = 2M + N.

It is apparent, that the most rapid changes in function K with time in the interval $0 \leqslant t \leqslant \tau$ will be determined by condition

$$\frac{dK}{dt} = A\cdot K, \qquad (17a)$$

i.e. when $K = \mathrm{const}\cdot\exp\int_0^t A\,dt$. Hence, considering (11) and (13) we obtain for $0 \leqslant t \leqslant \tau$: K = 0 and consequently

$$u \equiv 0 \text{ и } \theta \equiv 0. \qquad (17b)$$

Next we will do as follows. We will break up the values $t \geq 0$ into two classes. To the first class we will refer all t of the interval $0 \leq t \leq \gamma$, for which K = 0. The remaining t, for which $K \neq 0$, it is assumed are forming the second class. It is apparent, that the first class is not empty.

Such a breakdown, according to the Dedekind theorem, is justified and is done by the number γ, which appears to be maximum in first class.

In such an instance for all $t \leq \gamma$, $K(t)=0$ and for any ε exist such t', $0 < t'-\gamma < \varepsilon$, for which $K(t') \neq 0$.

On the other hand, it is evident from the proven facts with consideration of the continuity of function K(t) and condition $K(\gamma) = 0$, that for a certain ε_1 and all t, satisfying inequality $0 \leq t - \gamma \leq \varepsilon_1$, $K(t) = 0$, which contradicts the made assumption. Consequently, all values t are included in the first class and therefore

$$u \equiv 0 \text{ и } \theta \equiv 0 \text{ для всех } t \geq 0. \qquad (17c)$$

As to the pressure then as shown by equation (7) it is possible to synonymously determine the pressure gradient only.

3. The mentioned in point 2 proof of the soleness theory in the solution of the basic internal problem of free thermal convection of liquid can also be expanded for the case of an external problem [1]. This can be realized by an ordinary maximum transition from the finite zone into the infinite, requiring the fulfillment of conditions of dampint at infinity:

$$|r^{1+\alpha} v_i|, \quad |r^{1+\alpha} T_i|, \quad |r^{\alpha} p_i|, \quad |r^{1+\alpha} \operatorname{rot} v_i|, \quad |r^{1+\alpha} \operatorname{grad} T_i| \to 0 \quad (i = 1, 2), \qquad (17d)$$

where alpha $\geq \frac{1}{2}$, r - distance from a certain, assumed as origin of coordinates, point of heater.

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